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Topological Interlocking of Convex Regular Polyhedra

Vera Viana, CEAU - FAUP (Faculty of Architecture of Porto's University), Portugal

Introduction

Topological interlocked assemblies, in which a number of "elements (blocks) of special shape are arranged in such a way that the whole structure can be held together by a global peripheral constraint, while locally the elements are kept in place by kinematic constraints" (Estrin et al., 2011) are not a novelty in architectural design, but have known recent interesting developments. Glickman (1984) and Dyskin at al. (2003) demonstrated that a layer of equal tetrahedral blocks hold in place provided that each set of four is assembled in such a way that their half-section (with the edges' midpoints as vertices), outline a tessellation of four squares around each vertex (Fig. 1, upper left). From this research, other authors such as Dyskin et al. (2003), Kanel-Belov et al. (2008) and Weizmann et al. (2017) enlarged the repertoire of geometrical possibilities of topological interlocking (TI) to the remaining platonic solids and other polyhedral blocks.
Topological interlocked assemblies are able to withstand considerable external loads and tension, even without the use of binders or mortars if, for each situation, "rigid peripheral constraints to unfold load-bearing behaviour" are conceived (Tessman, 2012). Experimental researches have shown that these assemblies have an inherent toleration to the removal of isolated blocks without impairing the overall structure, a good acoustic performance, a thermal insulation and an enhanced seismic resistance, given that the kinematically constrained elements are already fragmented blocks (Tessman, 2013; Weizmann et al., 2017).

Research

In this research, we depart from a crossdisciplinary standpoint to address monohedral topologically interlocked systems as a mathematical problem (Viana, 2018) to illustrate the repertoire of geometrical possibilities for equal convex regular polyhedra to topologically interlock and expand those obtainable with pentagonal dodecahedra.
To understand the concept of topological interlocking (TI), we consider a plane that, sectioning a layer of interlocked polyhedra in half, outlines its "interlocking diagram" (Kanel-Belov, 2008). Each polyhedral block is interlocked by its neighbours if the polygons resulting from the translation of the section plane degenerate into a segment or a point (Dyskin, 2003) or into a finite region (as another polygon), as Kanel-Belov et al. later clarified, describing this procedure as an ''evolutionary transformation of the interlocking diagram'' (Kanel-Belov, 2008). Each of these situations are respectively distinguishable, in the TI illustrated in Fig. 1, of tetrahedra, whose cross-sections degenerate into edge-segments; cubes, icosahedra or dodecahedra (middle right), whose cross-sections degenerate into vertices; and octahedra or dodecahedra (bottom left), into polygons.
The interlocking diagram might, in certain situations, be a regular plane tessellation. In this regard, the concept of faceting (Coxeter, 1973) is quite useful to deduce which polygons are obtainable from each polyhedron and the tessellations they may outline.
If we use vertices of a given polyhedron to outline new polygons, we are faceting the polyhedron. These new polygons are its facets, which may themselves outline other polyhedra (as, for instance, eight equilateral triangles inside a cube outline two distinct tetrahedra or a 'stella octangula'). Ten regular facets are obtainable from convex regular polyhedra (Cromwell, 1997): from the cube, triangles; from the octahedron: squares; from the dodecahedron: two kinds of triangles, squares, pentagons and pentagrams; from the icosahedron: triangles, pentagons and pentagrams.
In addition to these, the half-section of the cube, the octahedron or the dodecahedron perpendicular to any 3-fold axis of rotational symmetry is a regular hexagon (a similar cross-section of the icosahedron is an equilateral decagon, though). The TI of cubes, octahedra and dodecahedra (following Dyskin, 2003) illustrated in Fig. 1 (middle right) are deducible from the regular tessellation outlined by their hexagonal cross-sections.
A plane perpendicular to any 5-fold symmetry axis of the dodecahedron or the icosahedron, passing through the midpoints of their edges, has a regular decagon for result (Holden, 1971). In the bottom row of Fig. 1, the TI of dodecahedra and icosahedra devised by Kanel-Belov et al. (2008) are illustrated. The concave quadrilateral gaps between the decagonal cross-sections have the least possible area, in order to allow coplanar faces of polyhedra to intersect in every decagons' side and every triad of polyhedra to interlock.
Although Dyskin et al. (2003) mention that no arrangements of polyhedra obtainable from a tessellation of triangles should be considered as interlocking assemblies, since no two pairs of faces of neighbouring blocks would exist to prevent each from moving upwards and downwards, we present (Fig. 2) an interesting topological interlocked assembly of pentagonal dodecahedra with parallel 3-fold symmetry axis, large enough to kinematically stand constrained with no gaps nor mutual intersections, in which the centroids describe a tessellation of equilateral triangles (Fig. 2, upper left). The half-section of dodecahedra are regular hexagons that tesselate the plane with equilateral triangles (Fig. 2, upper right). This interlocking diagram evolves into a tessellation of cyclic equiangular hexagons with equilateral triangles as gaps (Fig. 2, bottom). Three vertices of each of these hexagons coincide with the vertices of a triangular facet (the larger one) of each dodecahedron. For a better understanding of this interlocked assembly, Fig. 2 depicts also an orthographic view. This possibility has not been considered by the referenced authors and thus expands the repertoire of the geometrical possibilities of TI with platonic solids. Further research will confirm its effectiveness as an interlocked assembly and suggest other possibilities, to which the remaining convex uniform polyhedra will be considered.

Conclusion

Following a brief summary on the concept of topological interlocking, we introduced some of the outcomes of a systematic analysis of this concept from a mathematical viewpoint, that considered convex regular polyhedra as repetitive elements of topological interlocked assemblies. Through an approach that aimed to combine perspectives of two different fields of knowledge, seven geometrical possibilities of topological interlocked assemblies with the platonic solids have been illustrated, as potential leitmotif for further researches with possible applications in architectural design.

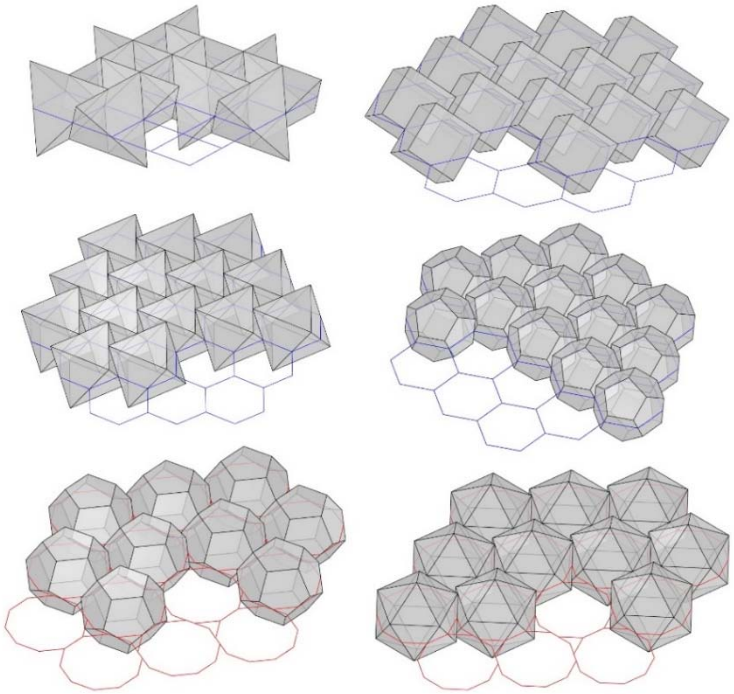

Fig. 1: Topological interlocking of convex regular polyhedra.

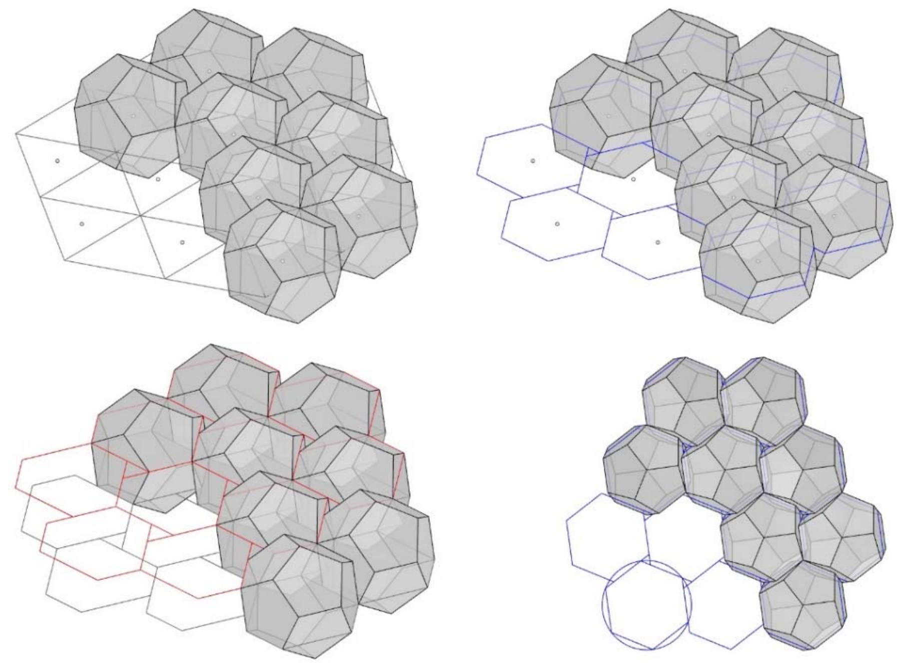

Fig. 2: Topological interlocking of pentagonal dodecahedra.

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Keywords

Topological Interlocking, convex regular polyhedra, architectural design

Biography

Vera Viana graduated in the Fine Arts Faculty of Porto's University, Portugal.
Ph.D. Student at UTAD (Trás-os-Montes and Alto Douro's University).
Integrated Researcher at CEAU/FAUP (Faculty of Architecture of Porto's University), focusing on polyhedral geometry and the relationships between architecture and mathematics.
Director of Aproged (Portuguese Geometry and Drawing Teachers' Association) and organizer of the International Conferences Geometrias. Editor-in-Chief of Aproged's Proceedings and Bulletins.
Conference Report Editor for the Nexus Network Journal.
As a teacher of descriptive geometry, teacher trainer, authorized Rhino Trainer and author of two schoolbooks on descriptive geometry, Viana is engaged in the development of educational resources with dynamic geometry, three-dimensional modelling and algorithmic modelling software, having authored papers and presentations on the subject.

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