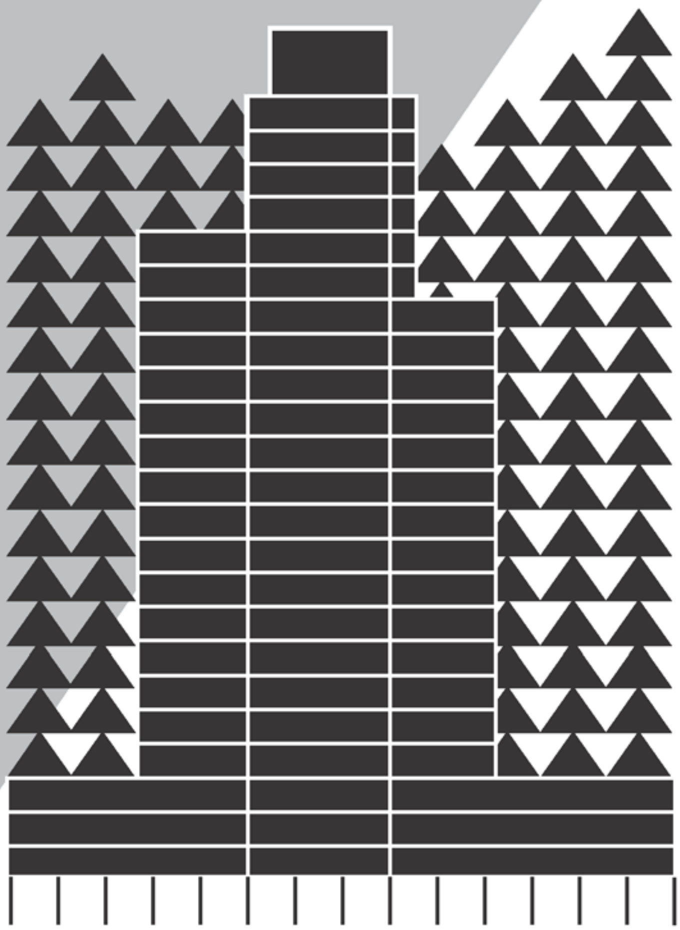
NEXUS 2021

Architecture and Mathematics

Conference Book



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CONTENTS

TOWARDS A SYSTEMATIC APPROACH OF TOPOLOGICAL INTERLOCKED ASSEMBLIES OF POLYHEDRA

Vera Viana[1]

Introduction

Topological interlocked assemblies (TIAs) are modular structures in which each solid block shares a portion of its surface with its neighbouring blocks in such a way that the whole assembly is held together with no mortars or adhesive elements. Blocks interlock with each other not only because of kinematic constraints deriving from their geometric configuration and position, but also for the peripheral restraint that encircles the whole structure, ensuring that everything is kept in its place.

In the 21st century, many solutions for TIA supported by computational design and digital fabrication have known extremely interesting solutions that challenge our perception on how architecture is able to materialize complex geometries. Some of the examples found in vaulted systems, domed spaces, or free-form surfaces would be interesting case-studies on their own, but the goal of our analysis is different. Rather than analysing TIA for their structural efficiency, we address the concept of topological interlocking as a geometric problem, aiming to clarify its definition, provide tools for its systematic study and, hopefully, inspire scholars, architects, engineers, and researchers to explore new solutions for its application. The present study focuses on monohedral TIA of convex uniform polyhedra. The notation TIA will be used to abbreviate topological interlocked assemblies; the initials in superscript identify which polyhedra are involved in each assembly.

The Research

The concept of topological interlocking

In 1984 Michael Glickman introduced the "vertically interlocking paving" in which his G-blocks, obtained from regular tetrahedra cross-sectioned perpendicularly to a 2-fold axis at equal distances from opposite edges, "pack beautifully and form a very stable array". Like any tetragonal disphenoid[2] these blocks are capable of interlocking and span planar structures, if the squares outlined by the midpoints of four edges[3] of each are faces of a regular tessellation (Fig. 1). Each square shares its centre with the corresponding tetrahedron, and so we name the tessellation they outline as midsection tessellation (MT). To avoid gaps or overlaps, all blocks must be placed in such a position that each pair has a portion of its faces in common (in this case, rhombi).

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[2] Tetragonal disphenoids have equal isosceles faces, equal solid angles (Coxeter 1973: 15) and are inscribable in cuboids.

[3] Glickman (1984) names these as equatorial squares of the tetrahedron. According to Coxeter (1973: 18), equatorial polygons are inscribed in great circles of the polyhedron's circumsphere, which is not the case here.

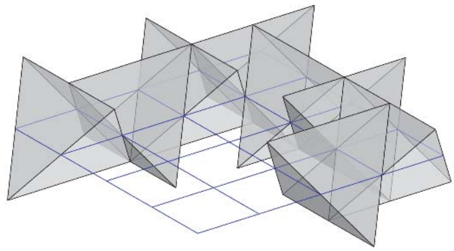

Fig. 1. TIA^T of regular tetrahedra, after Glickman (1984). (MT: squares)

In 2001, Dyskin, Estrin, Kanel-Belov and Pasternak, unaware of Abeille's or Glickman's investigations (according to Estrin 2014: 11), introduced the concept of "topological interlocking" with a layer of tetrahedron-shaped blocks (similar to Fig. 1) that could "withstand considerable loads even if no binder is used to hold the elements together" (Dyskin et al. 2001: 2694). In subsequent papers, Dyskin et al. (2003a, 2003b) explored the concept for the remaining convex regular polyhedra and introduced the principle of "cross-section evolution" (Dyskin et al. 2003a: 199), later designated as "evolutionary transformation" (Kanel-Belov et al. 2008: 10), through which the mid-section of the TIA is taken as sectioning plane. Translating it in the direction of its normal produces polygons in each polyhedron, that evolve into other configurations. The authors state that "An element is locked if, and only if, by continuously shifting the section plane in either direction the corresponding polygon eventually degenerates to a segment or a point". However, this is not the case for the interlocked octahedra in Dyskin et al. (2003a: 200), whose "evolutionary diagram" transforms from regular hexagons into equilateral triangles.

Aiming to establish the principles for TIA, Kanel-Belov et al. denote that polyhedra interlock if and only if "the corresponding polygon vanishes during the transformation", adding that the "polygon's evolution is exactly the region bounded by border planes of the corresponding polyhedron" (Kanel-Belov et al. 2008: 10). This would mean that, as finite regions, the polygonal cross-sections might evolve into polygons in the final stages. Given that this principle, formulated as it is, would still apply whether the layer is comprised of interlocked, tessellated or even distanced polyhedra, it is our belief that more specific conditions should be considered to define these assemblies.

Similar concerns are shared by Zhao Ma: "Research conducted that is related to TI system never state clearly why some polyhedra can be assembled into a TI system while some cannot … and what type of polyhedron allows TI in general, or whether there are systematic ways of assembling polyhedra into a TI system" (Ma 2017: 21).

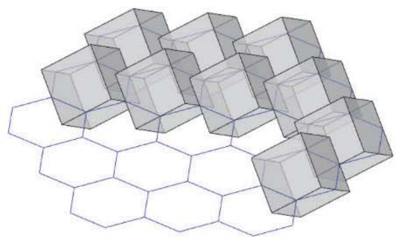

Fig. 2. TIA^C of cubes, after Dyskin et al. (2001) (MT: regular hexagons)

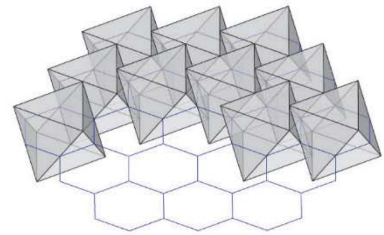

Fig. 3. TIA^O of regular octahedra, after Dyskin et al. (2001) (MT: regular hexagons)

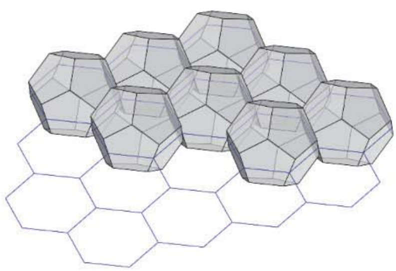

Fig. 4. TIA^D 1 of regular dodecahedra, after Dyskin et al. (2001) (MT: regular hexagons)

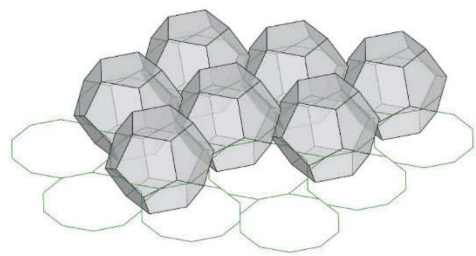

Fig. 5. TIA^D 2 of regular dodecahedra, after Dyskin et al. (2001) (MT: regular decagons and darts)

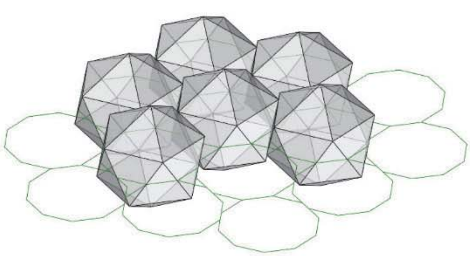

Fig. 6. TIA^I of regular icosahedra, after Dyskin et al. (2001) (MT: regular decagons and darts)

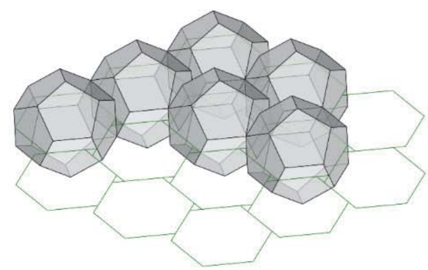

Fig. 7. TIA^D 3 of regular dodecahedra (Viana 2018b) (MT: regular hexagons and equilateral triangles)

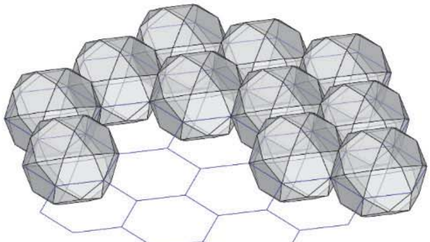

Fig. 8. TIA^{ID} of icosidodecahedra (Viana 2018a) (MT: regular hexagons)

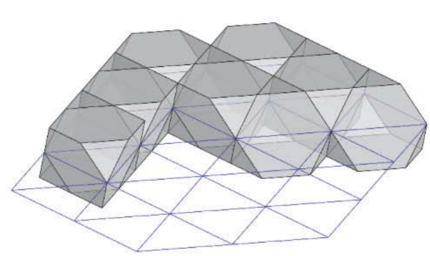

Fig. 9. TIA^{tT} 1 of truncated tetrahedra (MT: equilateral triangles)

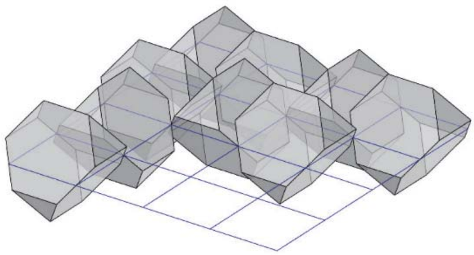

Fig. 10. TIA^{tT} 2 of truncated tetrahedra, based on TIA^{T} (MT: squares)

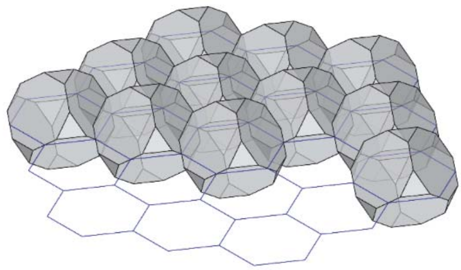

Fig. 11. TIA^{tC} of truncated cubes, based on TIA^{C} (MT: regular hexagons)

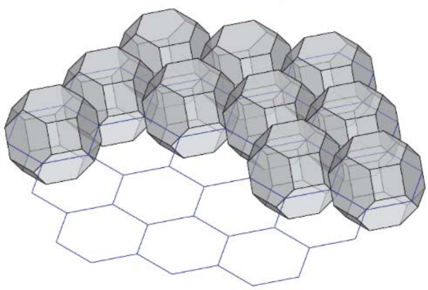

Fig. 12. TIA^{tO} of truncated octahedra, based on TIA^{O} (MT: regular hexagons)

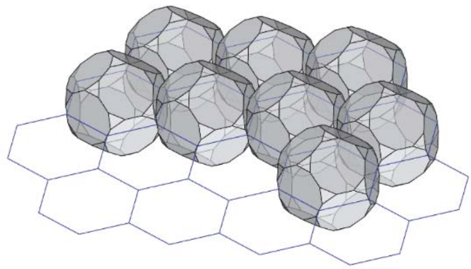

Fig. 13. TIA^{tD} 1 of truncated dodecahedra, based on TIA^{D} 1 (MT: regular hexagons)

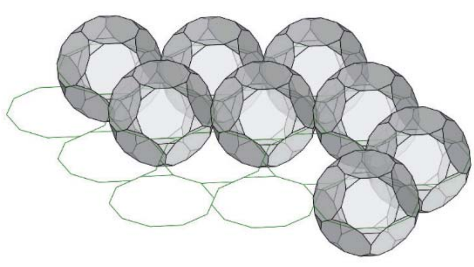

Fig. 14. TIA^{tD} 2 of truncated dodecahedra, based on TIA^{D} 2 (MT: regular decagons and darts)

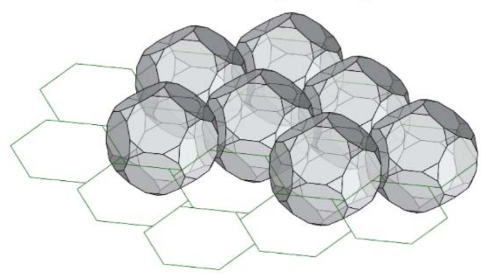

Fig. 15. TIA^{tD} 3 of truncated dodecahedra, based on TIA^{D} 3 (MT: regular hexagons and equilateral triangles)

Topological interlocking assemblies with convex uniform polyhedra

Dyskin et al. (2003a: 202) declare that the TIA they described for equal convex regular polyhedra, illustrated in Figures 1 to 6, "completes the catalogue of the interlocking arrangements for the platonic solids". Yet a detailed analysis (Viana 2018b) allowed us to find another possibility for pentagonal dodecahedra to interlock: $TIA^{D}3$, shown in Fig. 7, in which dodecahedra also have parallel 3-fold axis of rotational symmetry but are more densely packed than in $TIA^{D}2$. Figs. 8 to 15 illustrate our conclusions so far on TIA with the Archimedeans (for the sake of concision, we have not included TIA^{tI} based on TIA^{I}). Subsequent research will allow us to obtain more possibilities from this and other classes of polyhedra.

Conditions for polyhedra to topologically interlock

The following are introductory considerations on TIA of equal convex uniform polyhedra, that we hope additional research, supported by addictive manufacturing and parametric modelling, will substantiate. A crucial feature for TIA is mentioned by Dyskin et al., regarding any block of an assembly: “It is the inclined faces of [the] adjacent blocks that prevent the upward movement of the reference block. A similar argument holds for downward displacement which is blocked by the other two neighbours” (Dyskin et al. 2003a: 199). The same principle is discussed by Tessman: “The overall shape of an [interlocked] assembly emerges from the orientation of modules and their contact faces. … the interface planes of one module need to lock all rotational and transitional degrees of freedom” (Tessmann 2012: 30), and also by Pfeiffer et al. (2020: 2) and Zhao Ma (2017: 26-28). Hence, a necessary condition for TIA of polyhedra is that the orientation of the face planes surrounding each element blocks the degrees of freedom of that element. Similarly positioned, all neighbouring polyhedra interlock each other.

Equally crucial is that every TIA has the underlying plane tessellation, previously identified as MT, in which the centroid of the larger faces coincides with the centroid of the polyhedron from which they derive (Viana 2018a). Our research suggests that, for equal convex uniform polyhedra to interlock, the MT is not necessarily regular (as Weizmann et al. 2017 have shown); or even uniform (as in Figs. 5 and 6). In MTs that are not edge-to-edge tessellations of regular polygons (as in Fig. 7), the gaps between the larger polygons should have the least possible area to ensure the best interlocking performance between each triad of polyhedra. This also applies to TIA in Figs. 5 and 6.

Each pair of interlocked polyhedra must have a common plane in which faces share a common region. Research has pointed out that, to ensure efficiency in the system, these common regions should be polygons (in certain cases, faces) but not necessarily congruent, as in TIA^{I} (Fig. 6). In Dyskin et al. (2003b: 376), adjacent polyhedra in two examples do not seem to have polygons in common, which raises doubts about their interlocking efficiency. Without a more precise definition for the concept (including analysis of symmetric properties, packing density, structural efficiency, etc.) and subsequent categorization, assemblies of polyhedra that attend to some, but not all, of the aforesaid conditions, if given proper peripheral constraints, might be classifiable as TIA. Such could be the case of the “periodic stackings of dodecahedra” described by Koji Miyazaki (1986: 72) and some relief-structures conceived by the artist Gerard Caris, three of which match $TIA^{D}1$, $TIA^{D}2$ and $TIA^{D}3$, as pointed out by Cornelie Leopold (2018: 17).

In all the cases analysed so far, certain features come out as important, but a systematic research will help us certify their essentiality. Yet some open questions remain: Must equal polyhedra have parallel axis of rotational symmetry of equal order? Must all elements be convex? Can aperiodic TIA exist? Which TIA, not restricted to a planar development, are more effective? …

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