Architectonic Tessellations as Constructive Modules

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Vera Viana
University of Porto

NEXUS 2018
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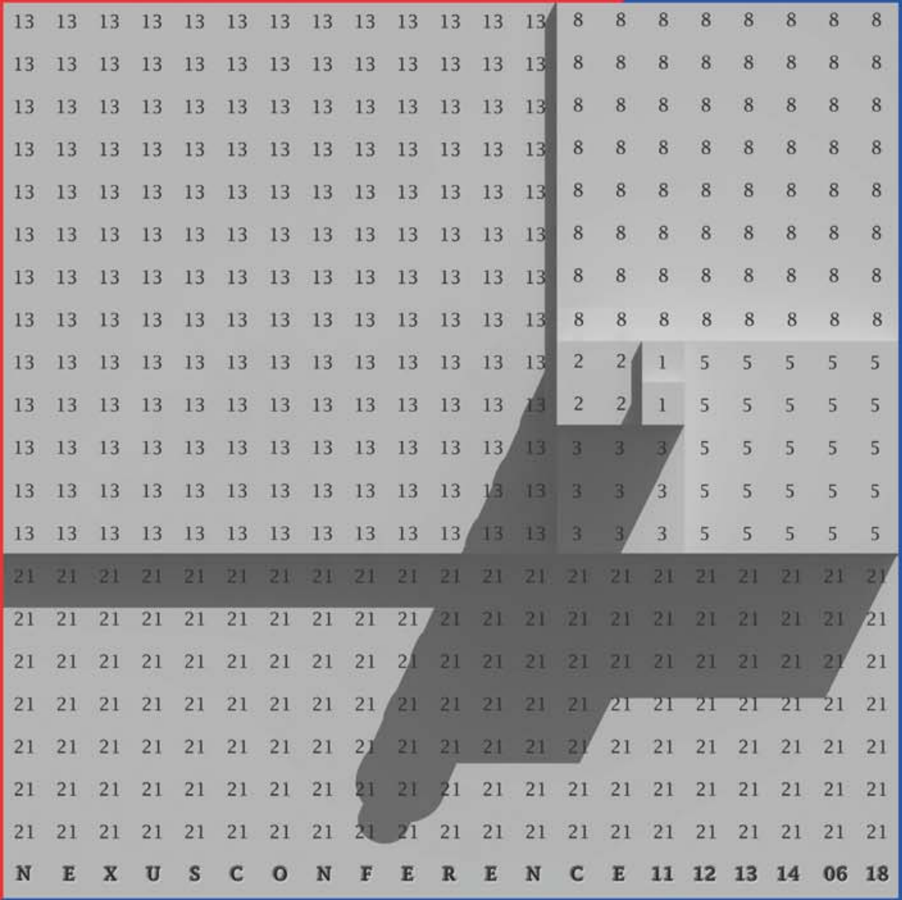

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ARCHITECTONIC TESSELLATIONS AS CONSTRUCTIVE MODULES

Vera Viana[1]

Introduction

All the cells in a uniform solid tessellation (or honeycomb) are uniform polyhedra that fit together to "fill all space just once, so that every face of each polyhedron belongs to one other polyhedron" (Coxeter 1973: 68). All the vertices are equally surrounded and superimposable under symmetries onto any other. The enumeration of polyhedra that fill space is an open problem in mathematics with "no finite answer" (Grünbaum & Shepard 1980: 966) but, if we restrain to convex regular-faced polyhedra, we conclude that only 28 possibilities in which space can be uniformly tessellated exist (Grünbaum, 1994: 49). A didactic experiment on the subject of 3D modelling with first-year students of architecture in 2017 developed into the exploration of solid tessellations as constructive modules. This presentation intends to illustrate some of its outcomes.

The Research

Theoretical framework

In 1905, Alfredo Andreini enumerated 23 possibilities to uniformly close-pack convex regular-faced polyhedra (which, in fact, were only 22, since one was mistakenly considered as uniform (Grünbaum 1994: 49)). Authors such as Keith Critchlow (1969), Robert Williams (1972, 1979) and Peter Jon Pearce (1978) addressed this theme but failed to consider the complete set of convex uniform honeycombs. Until the end of the 20th century, "mathematical literature was abundant with incomplete lists" of the uniform partitions on three-dimensional space (Deza & Shtogrin 2000: 1). The enumeration of convex uniform honeycombs would be accomplished only by Norman Johnson in 1991 and later confirmed by Branko Grünbaum (1994). 13 of these solid tessellations are considered as analogues to Archimedean plane tessellations and have been categorised by Conway, Burgiel and Goodman-Strauss (2008: 292) as "Architectonic tessellations, … because Architectonics is the theory of structural design and because its beginning reminds us of Archimedes". Given that these 13 (from which we exclude the tessellation outlined by stacks of equal cubes) are not merely piles of extruded planar tessellations and remain still relatively unexplored in architectural design, they were selected as leitmotiv for our didactic experiment with undergraduate students of architecture.

Examples of comparable polyhedral juxtapositions in architectural design (excluding the familiar cuboids) may be found in the works of Alexander Bell, Buckminster Fuller, Robert le Ricolais, Louis Khan, Jean-Francois Gabriel (all of whom explored

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the tessellation outlined by regular tetrahedra and octahedra); Zvi Hecker's *Synagogue in the Negev Desert* (1967-1969) that combines cuboctahedra, truncated octahedra and truncated tetrahedra; Giancarlo Mazzanti's *Forest of Hope* (2011) in which irregular truncated octahedra combine to define a structure resembling a canopy of trees; and, among others, Peter Jon Pearce's *Curved Space Structures (*1970s) that combine convex polyhedral forms in modular structures to serve as labyrinths in children's playgrounds. Interesting examples can also be found in the concept of topological interlocking developed by the Institute of Material Science, Technical University Clausthal, in which polyhedra and osteomorphic blocks interlock to outline mutually constrained planar configurations (Tessmann 2012: 1).

A teaching experiment with space-filling systems

The subject of polyhedra is one of the most interesting through which students might be introduced to 3D modelling software, given not only its importance for the development of students' spatial literacy, but, also, their tangibility as geometrical objects connected to several branches of knowledge. In this regard, Baracs (1998: 120) denotes that "if you can create an imagery which is movable, transformable, which you can manipulate, that is the best start for imagination and creation." Although this reference is addressed to physical models, we believe the same applies to their virtual counterpart, especially if students are given the opportunity to model polyhedra themselves from the ground-up, with the additional possibility of combining them in architectural projects as modular habitable spaces or structural spaceframes.

The interesting potential of uniform honeycombs as spatial structures, repetitive construction modules or stereotomic blocks and its absence from the *curricula* of the first year at the Faculdade de Arquitecura da Universidade do Porto led us to address this topic in 3 lectures, proposing students to model and manipulate virtual and physical models of uniform convex polyhedra. Subsequently, students were introduced to the concept of Voronoi cells and primary paralelohedra, some of which were modelled and combined in spatial arrays to outline convex honeycombs, so their potential as isogonal structural frameworks could be recognized.

The following task combined different sets of convex uniform polyhedra and the possibility of juxtaposing them face-to-face, to which the students' spatial skills and mental rotation abilities were particularly important. Subsequently, students became acquainted with the 28 convex uniform tessellations and conceived, in collaborative work, spatial modular structures for a specific location in *campus*. The idea was to creatively explore an architectonic tessellation and conceive a constructible modular composition, taking advantage, as Burry & Burry propose, of "the expressive potential of imperfection and of breaking rules often exploited in built work that draws on mathematical ideas of composition" (2012: 80). Figs. 1 and 2 depict examples of models from different groups of students.

This experiment stresses out the significance of polyhedral geometry in Architecture, Arts, Design and Engineering *curricula* and, in particular, the subject of space-filling polyhedra and spatial tessellations, and its contribution for the development of students' geometrical knowledge and spatial thinking.

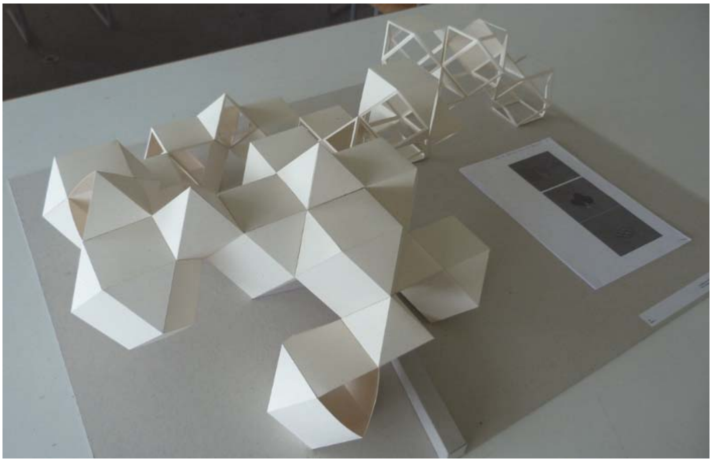

Fig. 1.

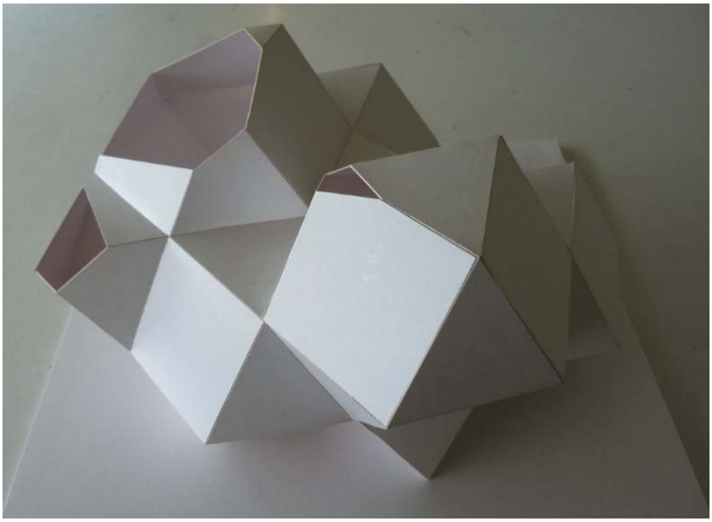

Fig. 2.

In this regard, the importance of a good 3D modelling software as pedagogical tool is not to be overlooked, without whom the possibility of operating “directly” in space to model polyhedra and combine them as modules would be less feasible or even impracticable. Further developments on this teaching experiment will propose students to conceive structural frameworks through algorithmic modelling, 3D printing and prototyping.

Conclusion

Our primary purpose with this investigation was not to introduce Students into polyhedra with the purpose of learning the 3D modelling software, but quite the contrary, to use the software as a strategy for students to learn polyhedral geometry. The procedures through which students solved the problems proposed allowed them to articulate their mathematical skills and geometrical knowledge, but further analysis of the work developed so far will allow us to understand if the spatial literacy of students advanced any further from the starting point.

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